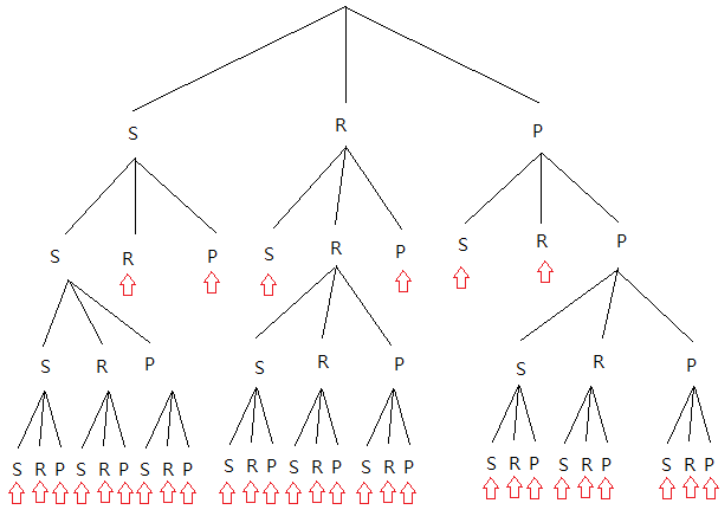
1. A palindrome is a bit string whose reversal is identical to the string. How many bit strings with length n are palindromes?

   $2^{\lceil \frac{n}{2} \rceil}$

2. Two people play rock-paper-scissors games, which ends when **one wins one game** or **two tie games occur**. Draw a tree diagram to illustrate how many ways the game can ends.



3. Prove that at a party where there are at least two people, there are two people who know the same number of other people there. (We assume that "knowing" is symmetric)

   Assume there are N people at the party.

   A person at the party can know 0 up to N-1 person at the party.

   However, if someone knows 0 person at the party, then no one at the party knows N-1 people at the party, and if someone knows N-1 people at the party, then no one knows 0 person at the party.

   Hence the range of number of people that one knows at the party have must be less than N.

   According to pigeonhole principle, there are two people who know the same number of other people there.

4. You are given 49 balls with 3 different colors. It is known that, for any 5 balls of the same color, at least two balls of them have the same weight. The 49

balls are distributed in two boxes. Prove that there are at least 3 balls which lie in the same box having the same color and also the same weight.

49 balls of three colors.

- ⇨ At least 17 balls have the same color.
- ⇨ At least 9 balls with the same colors in one box.

For any 5 balls of the same color, at least two balls of them have the same weight.

- ⇨ Balls with the same color have no more than 4 different weights.
- ⇨ At least 3 balls which lie in the same box having the same color and also the same weight.